Homework 10, M 331.2
due 12/7/16

Please hand in your home work before class, have it neatly written, organized (the grader will not decipher your notes), stapled, with your name and student ID on top.

Problem 1. Rewrite the 2nd order ODEs below as 2-dimensional systems of 1st order ODE , by introducing $v = y'$,

$$\begin{pmatrix} y \\ v \end{pmatrix}' = \begin{pmatrix} f_1(y,v) \\ f_2(y,v) \end{pmatrix}$$

Determine which of the ODEs is linear (homogeneous, inhomogeneous) and has constant coefficients. If the ODE is linear, use matrix notation and write it in the form

$$\begin{pmatrix} y \\ v \end{pmatrix}' = A(t) \begin{pmatrix} y \\ v \end{pmatrix} + \begin{pmatrix} r_1(t) \\ r_2(t) \end{pmatrix}$$

with $A(t)$ the 2x2 coefficient matrix and $\begin{pmatrix} r_1(t) \\ r_2(t) \end{pmatrix}$ the inhomogeneity.

(i) $y'' - \frac{2}{t^2} y = \ln t$
(ii) $y'' + \sin y = 0$
(iii) $y'' + 5y' - 10y = \cos(3t) + e^{2t}$
(iv) $y'' + y^2 + ty = 0$

Problem 2. Consider the matrix ODE

$$\vec{y}\,'(t) = \begin{pmatrix} -2 & 1 \\ 1 & -2 \end{pmatrix} \vec{y}(t)$$

where $\vec{y}(t) = \begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix}$.

(i) Find the eigensolutions $\vec{y_1}(t)$ and $\vec{y_2}(t)$.
(ii) Draw a phase plane picture showing the eigensolutions.
(iii) Find the solution to the initial condition $\vec{y}(0) = \begin{pmatrix} 0 \\ 4 \end{pmatrix}$ by superposition of the eigensolutions and draw the solution curve $\vec{y}(t)$.

Problem 3. Consider the matrix ODE

$$\vec{y}\,'(t) = \begin{pmatrix} -2 & 3 \\ 0 & 1 \end{pmatrix} \vec{y}(t)$$

where $\vec{y}(t) = \begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix}$.

(i) Find the eigensolutions $\vec{y_1}(t)$ and $\vec{y_2}(t)$.
(ii) Draw a phase plane picture showing the eigensolutions.
(iii) Find the solution to the initial condition $\vec{y}(0) = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ by superposition of the eigensolutions and draw the solution curve $\vec{y}(t)$.