Practice Exam, M 331.2
Fall 2016

Problem 1. Consider the ODE

$$y' = y(y-4)\,.$$

(i) Determine the equilibrium solutions and characterize them as stable, unstable or semi-stable.
(ii) Draw a slope line picture, indicate the equilibrium solutions, and sketch the overall behavior of the solutions.
(iii) Find the general solution of this ODE.
(iv) Find the solution with initial condition $y(0) = 1$.

Problem 2. Solve the ODE

$$y' + 5y = e^{2t}\cos(t)$$

with $y(0) = 0$.

Problem 3. Find the general solution to the ODE

$$y'' - 6y' + 9y = \frac{e^{3t}}{t}$$

Problem 4. Find the general solution to the ODE

$$y'' + 4y' + 4y = e^{-t}$$

Problem 5. Find the solution to the ODE

$$y'' + 2y' + 5y = \sin(t)$$

with initial condition $y(0) = 0$ and $y'(0) = 0$. If this ODE presents a harmonic oscillator, is it over/under or critically damped?

Problem 6. Consider the 2nd order ODE

$$y'' + 4y' + 4y = 0$$

(i) Rewrite the ODE as a 2-dimensional 1st order system.
(ii) Find the general solution to the system of ODEs.
(iii) Characterize whether the origin is a sink, source, saddle or spiral point.

Problem 7. Consider the ODE

$$\vec{y}\,' = \begin{pmatrix} 1 & 1 \\ -4 & 1 \end{pmatrix} \vec{y}$$

(i) Find the solution with initial condition $\vec{y}(0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.
(ii) Draw a picture of the general behavior of the solutions (in particular eigensolutions if there are any).
(iii) Characterize whether the origin is a source, sink, saddle or spiral point.

Problem 8. Consider the matrix ODE

$$\vec{y}\,' = \begin{pmatrix} 1 & 3 \\ 3 & 1 \end{pmatrix} \vec{y}$$

(i) Find the general solution.

(ii) Sketch the eigensolutions (if there are any) and sketch the general behavior of the solutions.
(iii) Characterize whether the origin is a source, sink, saddle or spiral point.