

HOMEWORK 1, M 331.2

DUE 9/19/16

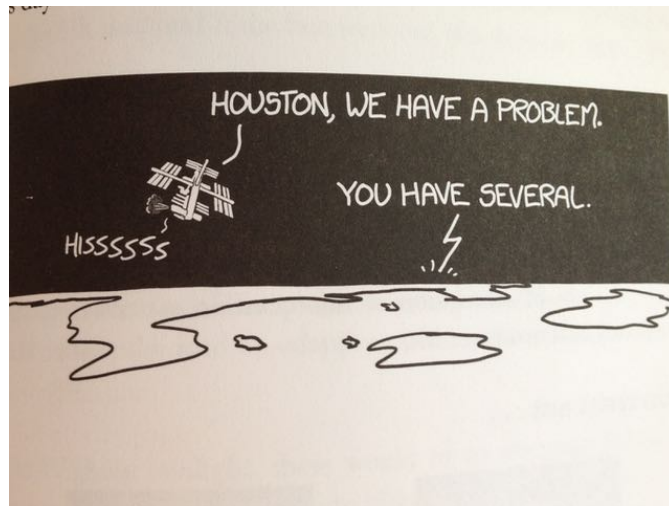
Please hand in your home work before class, have it neatly written, organized (the grader will not decipher your notes), stapled, with your name and student ID on top.

Problem 1. You are jumping off a 30 foot high diving board into a pool (ignore air resistance).

- (i) Write down the ODE describing this situation and solve it.
- (ii) Calculate the impact velocity when you reach the water.
- (iii) How long are you falling till you hit the water?

Problem 2. In a zero-gravity environment with an atmosphere identical to Earth's (e.g. in the ISS) you horizontally shoot an arrow of mass $m = 0.02$ kilograms with initial speed 75 meter/sec. Use the model that the air resistance is proportional to the square of the velocity of the arrow with drag coefficient $\gamma = 6 \times 10^{-5}$ kilogram/meter. Write down the ODE describing this situation and answer the following questions:

- (i) How far does the arrow fly in the first 5, 10, 30 seconds?
- (ii) What are the velocities of the arrow after 5, 10, 30 seconds?
- (iii) Will the arrow ever come to a standstill and hover in mid air or fly on forever? How far does the arrow fly without any obstacles?
- (iv) Assuming that an arrow of speed 7 m/s does not harm anymore, what is the shortest distance from you that your friend can stand without being harmed by being shot at?



Problem 3. A country road leading out of a village in the hills slopes (more or less at a constant grade) for a stretch of 850 meters downhill. The village wants to determine the grade of that stretch of road and put up a grade sign. The village's math teacher rolls a steel marble down the length of the road and records that it takes the marble 50 seconds. What is the grade of the road if we neglect friction effects?

Problem 4. Of course there will be some friction on the steel marble in the previous problem. Assuming that the force due to friction can be modeled to be proportional to the velocity of the steel marble, write down the ODE describing the situation. What data would you need in order to be able to determine all the coefficients in your ODE?

Problem 5. A mountaineer tries to cross a ridge, stumbles and slides down a 45 degree steep, 200 feet long, snow field. At the end of the snow field are rocks and boulders which the mountaineer is certain to hit. What is her impact velocity? Since outdoor clothes are fairly slippery, we can ignore friction.

Problem 6. How high does a vertically upwards thrown stone with an initial push of 5 meter/sec fly? (Assume the stone is launched from the ground and ignore air resistance).

Problem 7. The population dynamics of rabbits in a certain habitat is described by

$$\frac{dP}{dt} = P - 500$$

where $P(t)$ denotes the rabbit population at time t (measured in months), and 500 is the amount of rabbits eaten by preying animals etc per month.

- (i) For which initial rabbit population P_0 does the rabbit population stay constant over time?
- (ii) For which initial rabbit populations do the rabbits die out?
- (iii) Can it also happen that the rabbits just keep growing despite the fact that some are eaten all the time? What initial population is needed for that to happen?

Problem 8. A detective is called to the scene of a crime where a dead body has just been found. She arrives on the scene at 10:23 pm and begins her investigation. Immediately, the temperature of the body is taken and is found to be 80 F. The detective checks the programmable thermostat and finds that the room has been kept at a constant 68 F for the past 3 days. After evidence from the crime scene is collected, the temperature of the body is taken once more and found to be 78.5 F. This last temperature reading was taken exactly one hour after the first one. The next day the detective is asked by another investigator, “What time did our victim die”? Assuming that the victims body temperature was normal (98.6 F) prior to death, what is her answer to this question.

You may assume Newton’s Law of cooling applies: the rate of change of the temperature of the object is proportional to the difference between the object’s temperature and the ambient temperature. The proportionally factor is called the heat coefficient k . If you don’t know about Newton’s law of cooling, read it up in a book or google it.

Problem 9. On a hot day, while people are working inside a building, the air conditioner keeps the temperature at 24 C inside the building. At noon the air conditioner is turned off and the people go home. The outside temperature in the afternoon and through the evening is a constant 35 C. If the heat coefficient for the building is $k = 1/4$, what is the temperature inside the building at 2 pm? At 6 pm? When will the temperature in the building reach 27 C? Again, Newton’s law of cooling applies.

Problem 10 (Bonus Problem). In class we asked ourselves whether the ODE

$$y' = ky$$

where k is a constant, has any other solutions than $y(t) = ce^{kt}$ where c is some real number (to be determined by the initial condition). How would one go about this? Well, let $g(t)$ be that putative (perhaps different from e^{kt}) solution of $y' = ky$. Then we would have to convince ourselves that $g(t) = ce^{kt}$ for some real number c . Here is a hint how you could achieve this: consider (you may justifiably ask why, but this is where ingenuity/creativity/trial and error/playing with the problem etc comes in) the function

$$\frac{g(t)}{e^{kt}}$$

and try to show that this function is constant, i.e. independent of t . Recall a criterion from calculus how the derivative can be used to show that a function is constant and keep in mind that by assumption (which is important!) the function $g(t)$ is a solution of $y' = ky$