HOMEWORK 3, M 331
DUE 10/5/16

Please hand in your home work before class, have it neatly written, organized (the grader will not decipher your notes), stapled, with your name and student ID on top.

Problem 1. Which of the following first order ODEs are separable, linear (homogeneous, inhomogeneous), or neither? You don't need to solve any of the ODEs.

(i) $y' = y^2 + t$
(ii) $y' = \sin(t)e^y$
(iii) $y' = y^3 + 2y + 7$
(iv) $y' = (t^2 + 1)y + e^t$
(v) $y' = e^{-t}\cos(t)$
(vi) $y' = 2te^{y-t^2}$
(vii) $ty' = y$

Problem 2. Find the solution of the ODE

$$y' = \frac{1}{\ln y}$$

with initial condition $y(0) = 1$ (you may not be able to solve explicitly for y).

Problem 3. An object launched at an initial speed of $100\,m/s$ travels in a certain medium. We describe the motion by two models M1 and M2. Model 1 assumes that the drag on the object by the medium is proportional to velocity, and in model 2 proportional to the square of the velocity. We assume gravitation does not play a role (due to say buoyancy effects), then the equations of motion for the position $y(t)$ and velocity $v = y'$ are

$$v' = -v$$

for M1 and

$$v' = -v^2$$

for M2 (the drag coefficient per mass unit is 1).

(i) How long does it take in each model for the object to slow down to a velocity of $1\,m/s$?
(ii) How far does the object fly for each model until it has velocity $1\,m/s$?
(iii) What happens for large times, i.e., calculate the limits of $y(t)$ and $v(t)$ as $t \to \infty$ for both models.
(iv) Draw the accurate graphs of $y(t)$ for both models in the same plot. Do the same for the velocity function $v(t)$ for both models.
(v) Which model has the smaller drag force for small velocities (say below $1\,m/s$)? Which model has the smaller drag force for large velocities?

Problem 4. For the ODEs

(i) $y' = \frac{1}{y}$
(ii) $y' = \frac{1}{t^2 \sin y}$
(iii) $y' = \sqrt{y^2 - 4}\ln t$
(iv) $ty' + \ln y + y^2 = 0$
(v) $y' = e^t \frac{y}{y^2+y-2}$

determine the largest region D in the (t, y)-plane to which you can apply (our version of) the Fundamental Theorem of ODEs: if $y' = F(t, y)$ and $F(t, y)$ is differentiable in both variables then to any initial condition $(t_0, y_0) \in D$ there exists a unique solution $y(t)$ of the ODE with $y(t_0) = y_0$ for small time $t \in (t_0 - \epsilon, t_0 + \epsilon)$ for some $\epsilon > 0$. You do not need to solve the ODEs.

Problem 5. Consider the linear (inhomogeneous) ODE

$$y' + y = \sin t$$

(i) Find all solutions to the homogeneous ODE.
(ii) Find one particular solution of the inhomogeneous ODE.
(iii) Write down all solutions of the ODE.
(iv) What happens to the solutions when $t \to \infty$?
(v) Find the solution which satisfies $y(0) = 0$.

Problem 6. Find the solution to $ty' + 2y = t^2 - t + 1$ with initial condition $y(1) = 1/2$.

Problem 7. Find all solutions of the ODE $y' + 2ty = 2te^{-t^2}$.

Problem 8. Solve the following ODEs (if an initial condition is given, find the solution satisfying this condition):

(i) $y' - 2y = t^2 e^{2t}\,, \qquad y(0) = 0$
(ii) $ty' + (t+1)y = t\,, \qquad y(\ln 2) = 1$
(iii) $y' + y = \frac{1}{1+e^t}$