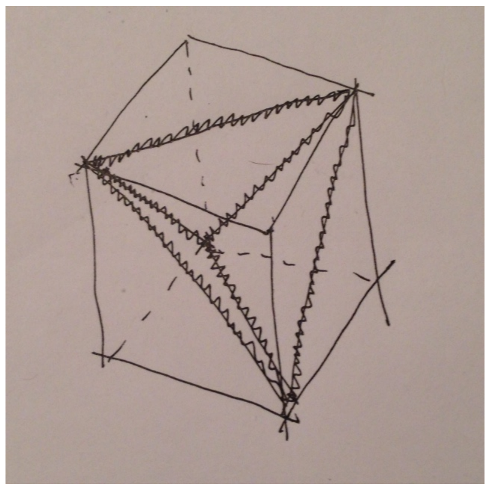
HOMEWORK 1, ADVANCED CALCULUS
DUE 2/15/17

Please hand in your home work before class, have it neatly written, organized (the grader will not decipher your notes), stapled, with your name and student ID on top.

Problem 1. Show that the polyhedron inscribed in the cube as shown in the (feeble) picture is a tetrahedron.



Problem 2. For each of the functions below calculate the directional derivative at the given point $\mathring{x}$ in the direction of v:

(i) $f(x_1, x_2) = (x_1 - x_2)^2$, $\mathring{x} = (1, 1)$, $v = (2, 3)$
(ii) $f(x_1, x_2) = x_1^3 - x_2$, $\mathring{x} = (1, 0)$, $v = (1, 1)$
(iii) $f(x_1, x_2, x_3) = x_1^2 + x_2^2$, $\mathring{x} = (1, 1, 2)$, $v = (2, 3, 4)$
(iv) $f(x_1, x_2) = x_1 x_2$, $\mathring{x} = (0, 1)$, $v = (1, 0)$
(v) $f(x_1, x_2, x_3) = x_1^2 - x_2^2 - x_3$, $\mathring{x} = (1, 0, 1)$, $v = (0, 1, 2)$

Problem 3. Find a general formula (some expression in x and v) for the directional derivative at the point $x = (x_1, x_2)$ in the direction of $v = (v_1, v_2)$ of the function

$$f(x_1, x_2) = \frac{x_1^3 x_2}{x_1^4 + x_2^2}$$

on the domain $\mathbb{R}^2 \setminus \{(0, 0)\}$. Use your formula to calculate the directional derivative at $x = (1, 1)$ in the direction $v = (1, 0)$.

Problem 4. In a cube of side length 10 with center at the origin in $\mathbb{R}^3$ the temperature distribution is given by the function

$$T(x_1, x_2, x_3) = 65 + \cos(x_1^2 + x_2^2 + x_3^2)$$

Calculate the rates of change of T per unit length at the origin in the directions of the two diagonals across the interior of the cube.

Problem 5. Calculate all the partial derivatives at an arbitrary point x for the functions below:

(i) $f(x_1, x_2) = x_1^2 - x_2^2$
(ii) $f(x_1, x_2) = x_1 + x_2$
(iii) $f(x_1, x_2) = x_1^3$
(iv) $f(x_1, x_2) = \cos(x_1^2 + x_2^2)$
(v) $f(x_1, x_2) = \frac{1}{x_1 - x_2}$ for $x_1 \neq x_2$

Problem 6. Find a formula for the directional derivative at $0 \neq x \in \mathbb{R}^n$ in direction $v \in \mathbb{R}^n$ for the function $f(x) = ||x||$. Try to express your answer using only the letters x, v, the dot product $<,>$ and $|| \, ||$.

Problem 7. Calculate the Jacobi matrix for the functions

(i) $f\colon \mathbb{R}^3 \to \mathbb{R}^2$ given by $f(x_1, x_2, x_3) = (x_1 x_2 x_3, x_1 + x_2 + x_3)$
(ii) $f\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $f(x_1, x_2, x_3) = (\sin(x_1), \cos(x_2), e^{x_3})$