HOMEWORK 4, M 331.2
DUE 10/12/16

Please hand in your home work before class, have it neatly written, organized (the grader will not decipher your notes), stapled, with your name and student ID on top.

Problem 1. At 5 pm on a Friday an office building has an inside temperature of 72 F. The heating/cooling system gets switched off over the weekend. Assume the outside temperature over the weekend varies sinusoidally with a maximum temperature of 85 F at 3 pm and a minimum temperature of 45 F at 3 am around an average temperature of 65 F. The heat coefficient for the building in question is $k = 1/2$.

(i) Use Newton's Law of cooling (rate of change of inside temperature $T(t)$ is proportional – via k – to the difference of external $E(t)$ an internal $T(t)$ temperatures) to write down the differential equation for this model (time t has unit hours).
(ii) Write down the solution of your ODE describing the temperature $T(t)$ inside the building at time t (in hours).
(iii) What is the highest and lowest temperature in the building over the weekend (including Sunday night).
(iv) At what times do the lowest and highest temperatures happen? When does the highest/lowest outside temperature happen?
(v) Is there a time lag between say the highest inside and outside temperature, and if so, how much is the time lag?

Problem 2. The concentration $C(t)$ of a drug inside the bloodstream changes over time t by the following model: the body eliminates the orally taken drug proportionally to the concentration $C(t)$ present at time t with an elimination factor $k = 0.4$. The body absorbs the drug into the blood stream via Ae^{-rt} with an absorption rate $r = 0.5$ and an absorption constant $A = 53$.

(i) Write down the corresponding ODE for this model.
(ii) Write down the solution of your ODE describing the concentration $C(t)$ in the blood stream at time t.
(iii) At which time after taking the drug is the concentration maximal?
(iv) At which time after taking the drug does the concentration reach 10% of its maximum value (at this point the patient is advised to take another pill).

Problem 3. Consider the linear (inhomogeneous) ODE

$$y' + 3y = \sin t + 2\cos t$$

(i) Find all solutions $y_H(t)$ of the homogeneous ODE.
(ii) Find a particular $y_P(t)$ solution of the inhomogeneous ODE.
(iii) Write down the general solution $y(t)$ of the ODE.
(iv) What happens to the solution $y(t)$ for large time?
(v) Find the solution which satisfies $y(0) = 4$. Draw an accurate graph of this solution (copy graph from a graphing calculator, indicate scales etc) for $t \geq 0$ and compare this to (iv).

Problem 4. Find the solution to $ty' + 2y = t^2 - t + 1$ with initial condition $y(1) = 1/2$.

Problem 5. Find the general solution of the ODE $y' + 2ty = 2te^{-t^2}$.

Problem 6. Is there number y_0 so that the solution $y(t)$ of the ODE

$$y' - y = 1 + \sin t$$

with initial condition $y(0) = y_0$ remains finite for all times?

Problem 7. Solve the ODE

$$y' + y = \frac{1}{1 + e^t}\,, \qquad y(0) = 0$$

Problem 8. Find the general solution of the ODE $y' + 5y = 2e^{5t}$.

Problem 9. Find the general solution of the ODE $y' - y = t\sin(t) + e^{2t}$.

Note that this is an ODE whose $p(t) = -1$ is constant, so you could – as in some of the previous problems – use the undetermined coefficient method. Since the inhomogeneity $r(t) = r_1(t) + r_2(t)$ is a sum of two functions, you add together your choices of particular solutions for each of the inhomogeneities to obtain a particular solution for the ODE.

Problem 10. After getting your degree you start a job paying 100 K a year. A certain investment account offers a 5% annual return. You decide to annually put 30 K into that account and you start out with nothing in the account.

(i) Write down the ODE for that situation.
(ii) Draw an accurate graph (labels, units) of the solution.
(iii) How much money will you have in the account after 5, 10 and 20 years? Apply common sense to check if the values are reasonable (compare with your graph).