HOMEWORK 7, M 331.2
DUE 11/1/16

Please hand in your home work before class, have it neatly written, organized (the grader will not decipher your notes), stapled, with your name and student ID on top.

Problem 1. Consider the linear 2nd order homogeneous ODE

$$y'' + \frac{3}{2t}y' - \frac{1}{2t^2}y = 0\,.$$

for $t > 0$. Note that this ODE does not have constant coefficients.

(i) Verify that $y_1(t) = \sqrt{t}$ and $y_2(t) = 1/t$ are solutions.
(ii) Calculate the Wronskian of the above two solutions at time $t_0 = 1$. Are those two solutions independent?
(iii) Calculate fundamental solutions for the initial time $t_0 = 1$.
(iv) Calculate the solution of the ODE to initial data $y(1) = -2$ and $y'(1) = 3$.

Problem 2. Consider the ODE

$$y'' - 2y' + y = 0\,.$$

(i) Calculate two independent solutions $y_1(t)$, $y_2(t)$ for this ODE.
(ii) Calculate fundamental solutions for the initial time $t_0 = 0$.
(iii) Calculate the solution of the ODE to initial data $y(0) = 1$ and $y'(0) = 1$.

Problem 3. Consider the non-constant coefficient ODE

$$y'' - \frac{t+2}{t}y' + \frac{t+2}{t^2}y = 0$$

for $t > 0$.

(i) Verify that $y_1(t) = t$ and $y_2(t) = te^t$ are solutions.
(ii) Calculate the Wronskian of the above two solutions at time $t_0 = 1$. Are those two solutions independent?
(iii) Calculate the solution of the ODE to initial data $y(1) = 0$ and $y'(1) = 2$.

Problem 4. Consider the homogeneous ODE $y'' + 6y' + 13y = 0$. From HW 6 we know that $y_1(t) = e^{-3t}\sin 2t$ and $y_2(t) = e^{-3t}\cos 2t$ are solutions of this ODE.

(i) Calculate the Wronskian of $y_1(t)$ and $y_2(t)$ and verify that these solutions are independent.
(ii) Find a fundamental system of solutions for the initial time $t_0 = 0$.
(iii) Calculate the solution with initial data $y(0) = -1$ and $y'(0) = 1$.

Problem 5. Calculate the Wronskian of the following ODEs (note that we gave a formula for the Wronskian in class which does not involve knowledge of solutions).

(i) $y'' + \frac{1}{t}y' + \frac{t^2-4}{t^2}y = 0$
(ii) $y'' - \frac{2t}{1-t^2}y' + \frac{4}{1-t^2}y = 0$
(iii) $y'' + 2y' + 5y = 0$

Problem 6. Let $z = 4 + 3i$ and $w = -1 + 2i$ be complex numbers.

(i) Calculate $z + w$, z^2, zw, $\frac{z}{w}$, $\frac{w}{z}$, $\bar{z}$ and express all results in the usual form $x + iy$.
(ii) Write z and w in polar form $r(\cos\theta + i\sin\theta)$ and calculate the corresponding r and θ .

(iii) Find the zeros λ_1 and λ_2 of

$$\lambda^2 + 2\lambda + 10 = 0$$

and verify that $\lambda_2 = \overline{\lambda_1}$.

Problem 7. Write the following complex numbers z in standard form $x + iy$:

$$z = e^{2+i}\,, \quad z = e^{i\pi}\,, \quad z = e^{-2+\frac{\pi i}{2}}\,, \quad z = 2^i$$