



PRINCETON UNIVERSITY PRESS

Book Search:

GO

Google contents of this website:

GO

Google full text of our books:

GO

ONLINE BOOK SALE

New Book E-Mails

New In Print

PUP Blog

Subjects

Catalogs

Series

Sample Chapters

Podcasts/Vodcasts

Recent Awards

E-Books

Online Books

Online Ordering

For Reviewers

Permissions

Class Use

About Us

Contact Us

European Office

Links

F.A.Q.

MATH SITE

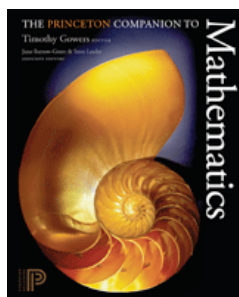
PHYSICS AND

ASTRONOMY

SITE

EINSTEIN SITE

PUP Home

**The Princeton Companion to Mathematics**

Edited by Timothy Gowers

June Barrow-Green and Imre Leader, associate editors

Book Description | Reviews

Preface [in PDF format] | Contributors [PDF only] | Article I.2 [PDF only] | Article II.2 [PDF only] | Article IV.21 [PDF only] | Article IV.5 [PDF only] | Article V.10 [PDF only] | Article VI-61 [PDF only] | Article VII-2 [PDF only] | Article VIII-6 [PDF only] | Table of Contents

TABLE OF CONTENTS:

Preface ix

Contributors xvii

Part I Introduction

I.1 What Is Mathematics About? 1

I.2 The Language and Grammar of Mathematics 8

I.3 Some Fundamental Mathematical Definitions 16

I.4 The General Goals of Mathematical Research 48

Part II The Origins of Modern Mathematics

II.1 From Numbers to Number Systems 77

II.2 Geometry 83

II.3 The Development of Abstract Algebra 95

II.4 Algorithms 106

II.5 The Development of Rigor in Mathematical Analysis 117

II.6 The Development of the Idea of Proof 129

II.7 The Crisis in the Foundations of Mathematics 142

Part III Mathematical Concepts

III.1 The Axiom of Choice 157

III.2 The Axiom of Determinacy 159

III.3 Bayesian Analysis 159

III.4 Braid Groups 160

III.5 Buildings 161

III.6 Calabi-Yau Manifolds 163

III.7 Cardinals 165

III.8 Categories 165

III.9 Compactness and Compactification 167

III.10 Computational Complexity Classes 169

III.11 Countable and Uncountable Sets 170

III.12 C^* -Algebras 172

III.13 Curvature 172

III.14 Designs 172

III.15 Determinants 174

III.16 Differential Forms and Integration 175

III.17 Dimension 180

III.18 Distributions 184

III.19 Duality 187

III.20 Dynamical Systems and Chaos 190

III.21 Elliptic Curves 190

III.22 The Euclidean Algorithm and Continued Fractions 191

III.23 The Euler and Navier-Stokes Equations 193

III.24 Expanders 196

III.25 The Exponential and Logarithmic Functions 199

III.26 The Fast Fourier Transform 202

III.27 The Fourier Transform 204

III.28 Fuchsian Groups 208

III.29 Function Spaces 210

III.30 Galois Groups 213

III.31 The Gamma Function 213

III.32 Generating Functions 214

III.33 Genus 215

III.34 Graphs 215

III.35 Hamiltonians 215

III.36 The Heat Equation 216

III.37 Hilbert Spaces 219

III.38 Homology and Cohomology 221

III.39 Homotopy Groups	221
III.40 The Ideal Class Group	221
III.41 Irrational and Transcendental Numbers	222
III.42 The Ising Model	223
III.43 Jordan Normal Form	223
III.44 Knot Polynomials	225
III.45 K-Theory	227
III.46 The Leech Lattice	227
III.47 L-Functions	228
III.48 Lie Theory	229
III.49 Linear and Nonlinear Waves and Solitons	234
III.50 Linear Operators and Their Properties	239
III.51 Local and Global in Number Theory	241
III.52 The Mandelbrot Set	244
III.53 Manifolds	244
III.54 Matroids	244
III.55 Measures	246
III.56 Metric Spaces	247
III.57 Models of Set Theory	248
III.58 Modular Arithmetic	249
III.59 Modular Forms	250
III.60 Moduli Spaces	252
III.61 The Monster Group	252
III.62 Normed Spaces and Banach Spaces	252
III.63 Number Fields	254
III.64 Optimization and Lagrange Multipliers	255
III.65 Orbifolds	257
III.66 Ordinals	258
III.67 The Peano Axioms	258
III.68 Permutation Groups	259
III.69 Phase Transitions	261
III.70 p	261
III.71 Probability Distributions	263
III.72 Projective Space	267
III.73 Quadratic Forms	267
III.74 Quantum Computation	269
III.75 Quantum Groups	272
III.76 Quaternions, Octonions, and Normed Division Algebras	275
III.77 Representations	279
III.78 Ricci Flow	279
III.79 Riemann Surfaces	282
III.80 The Riemann Zeta Function	283
III.81 Rings, Ideals, and Modules	284
III.82 Schemes	285
III.83 The Schrödinger Equation	285
III.84 The Simplex Algorithm	288
III.85 Special Functions	290
III.86 The Spectrum	294
III.87 Spherical Harmonics	295
III.88 Symplectic Manifolds	297
III.89 Tensor Products	301
III.90 Topological Spaces	301
III.91 Transforms	303
III.92 Trigonometric Functions	307
III.93 Universal Covers	309
III.94 Variational Methods	310
III.95 Varieties	313
III.96 Vector Bundles	313
III.97 Von Neumann Algebras	313
III.98 Wavelets	313
III.99 The Zermelo-Fraenkel Axioms	314
Part IV Branches of Mathematics	
IV.1 Algebraic Numbers	315
IV.2 Analytic Number Theory	332
IV.3 Computational Number Theory	348
IV.4 Algebraic Geometry	363
IV.5 Arithmetic Geometry	372
IV.6 Algebraic Topology	383
IV.7 Differential Topology	396
IV.8 Moduli Spaces	408
IV.9 Representation Theory	419
IV.10 Geometric and Combinatorial Group Theory	431
IV.11 Harmonic Analysis	448
IV.12 Partial Differential Equations	455
IV.13 General Relativity and the Einstein Equations	483
IV.14 Dynamics	493
IV.15 Operator Algebras	510
IV.16 Mirror Symmetry	523
IV.17 Vertex Operator Algebras	539
IV.18 Enumerative and Algebraic Combinatorics	550
IV.19 Extremal and Probabilistic Combinatorics	562
IV.20 Computational Complexity	575
IV.21 Numerical Analysis	604
IV.22 Set Theory	615
IV.23 Logic and Model Theory	635
IV.24 Stochastic Processes	647
IV.25 Probabilistic Models of Critical Phenomena	657

IV.26 High-Dimensional Geometry and Its Probabilistic Analogues 670

Part V Theorems and Problems

- V.1 The ABC Conjecture 681
- V.2 The Atiyah-Singer Index Theorem 681
- V.3 The Banach-Tarski Paradox 684
- V.4 The Birch-Swinnerton-Dyer Conjecture 685
- V.5 Carleson's Theorem 686
- V.6 The Central Limit Theorem 687
- V.7 The Classification of Finite Simple Groups 687
- V.8 Dirichlet's Theorem 689
- V.9 Ergodic Theorems 689
- V.10 Fermat's Last Theorem 691
- V.11 Fixed Point Theorems 693
- V.12 The Four-Color Theorem 696
- V.13 The Fundamental Theorem of Algebra 698
- V.14 The Fundamental Theorem of Arithmetic 699
- V.15 Gödel's Theorem 700
- V.16 Gromov's Polynomial-Growth Theorem 702
- V.17 Hilbert's Nullstellensatz 703
- V.18 The Independence of the Continuum Hypothesis 703
- V.19 Inequalities 703
- V.20 The Insolubility of the Halting Problem 706
- V.21 The Insolubility of the Quintic 708
- V.22 Liouville's Theorem and Roth's Theorem 710
- V.23 Mostow's Strong Rigidity Theorem 711
- V.24 The P versus NP Problem 713
- V.25 The Poincaré Conjecture 714
- V.26 The Prime Number Theorem and the Riemann Hypothesis 714
- V.27 Problems and Results in Additive Number Theory 715
- V.28 From Quadratic Reciprocity to Class Field Theory 718
- V.29 Rational Points on Curves and the Mordell Conjecture 720
- V.30 The Resolution of Singularities 722
- V.31 The Riemann-Roch Theorem 723
- V.32 The Robertson-Seymour Theorem 725
- V.33 The Three-Body Problem 726
- V.34 The Uniformization Theorem 728
- V.35 The Weil Conjectures 729

Part VI Mathematicians

- VI.1 Pythagoras (ca. 569 B.C.E.-ca. 494 B.C.E.) 733
- VI.2 Euclid (ca. 325 B.C.E.-ca. 265 B.C.E.) 734
- VI.3 Archimedes (ca. 287 B.C.E.-212 B.C.E.) 734
- VI.4 Apollonius (ca. 262 B.C.E.-ca. 190 B.C.E.) 735
- VI.5 Abu Ja'far Muhammad ibn Musa al-Khwarizmi (800-847) 736
- VI.6 Leonardo of Pisa (known as Fibonacci) (ca. 1170-ca. 1250) 737
- VI.7 Girolamo Cardano (1501-1576) 737
- VI.8 Rafael Bombelli (1526-after 1572) 737
- VI.9 François Viète (1540-1603) 737
- VI.10 Simon Stevin (1548-1620) 738
- VI.11 René Descartes (1596-1650) 739
- VI.12 Pierre Fermat (160?-1665) 740
- VI.13 Blaise Pascal (1623-1662) 741
- VI.14 Isaac Newton (1642-1727) 742
- VI.15 Gottfried Wilhelm Leibniz (1646-1716) 743
- VI.16 Brook Taylor (1685-1731) 745
- VI.17 Christian Goldbach (1690-1764) 745
- VI.18 The Bernoullis (fl. 18th century) 745
- VI.19 Leonhard Euler (1707-1783) 747
- VI.20 Jean Le Rond d'Alembert (1717-1783) 749
- VI.21 Edward Waring (ca. 1735-1798) 750
- VI.22 Joseph Louis Lagrange (1736-1813) 751
- VI.23 Pierre-Simon Laplace (1749-1827) 752
- VI.24 Adrien-Marie Legendre (1752-1833) 754
- VI.25 Jean-Baptiste Joseph Fourier (1768-1830) 755
- VI.26 Carl Friedrich Gauss (1777-1855) 755
- VI.27 Siméon-Denis Poisson (1781-1840) 757
- VI.28 Bernard Bolzano (1781-1848) 757
- VI.29 Augustin-Louis Cauchy (1789-1857) 758
- VI.30 August Ferdinand Möbius (1790-1868) 759
- VI.31 Nicolai Ivanovich Lobachevskii (1792-1856) 759
- VI.32 George Green (1793-1841) 760
- VI.33 Niels Henrik Abel (1802-1829) 760
- VI.34 János Bolyai (1802-1860) 762
- VI.35 Carl Gustav Jacob Jacobi (1804-1851) 762
- VI.36 Peter Gustav Lejeune Dirichlet (1805-1859) 764
- VI.37 William Rowan Hamilton (1805-1865) 765
- VI.38 Augustus De Morgan (1806-1871) 765
- VI.39 Joseph Liouville (1809-1882) 766
- VI.40 Eduard Kummer (1810-1893) 767
- VI.41 Évariste Galois (1811-1832) 767
- VI.42 James Joseph Sylvester (1814-1897) 768
- VI.43 George Boole (1815-1864) 769
- VI.44 Karl Weierstrass (1815-1897) 770
- VI.45 Pafnuty Chebyshev (1821-1894) 771
- VI.46 Arthur Cayley (1821-1895) 772
- VI.47 Charles Hermite (1822-1901) 773
- VI.48 Leopold Kronecker (1823-1891) 773

VI.49 Georg Friedrich Bernhard Riemann (1826-1866)	774
VI.50 Julius Wilhelm Richard Dedekind (1831-1916)	776
VI.51 Émile Léonard Mathieu (1835-1890)	776
VI.52 Camille Jordan (1838-1922)	777
VI.53 Sophus Lie (1842-1899)	777
VI.54 Georg Cantor (1845-1918)	778
VI.55 William Kingdon Clifford (1845-1879)	780
VI.56 Gottlob Frege (1848-1925)	780
VI.57 Christian Felix Klein (1849-1925)	782
VI.58 Ferdinand Georg Frobenius (1849-1917)	783
VI.59 Sofya (Sonya) Kovalevskaya (1850-1891)	784
VI.60 William Burnside (1852-1927)	785
VI.61 Jules Henri Poincaré (1854-1912)	785
VI.62 Giuseppe Peano (1858-1932)	787
VI.63 David Hilbert (1862-1943)	788
VI.64 Hermann Minkowski (1864-1909)	789
VI.65 Jacques Hadamard (1865-1963)	790
VI.66 Ivar Fredholm (1866-1927)	791
VI.67 Charles-Jean de la Vallée Poussin (1866-1962)	792
VI.68 Felix Hausdorff (1868-1942)	792
VI.69 Élie Joseph Cartan (1869-1951)	794
VI.70 Emile Borel (1871-1956)	795
VI.71 Bertrand Arthur William Russell (1872-1970)	795
VI.72 Henri Lebesgue (1875-1941)	796
VI.73 Godfrey Harold Hardy (1877-1947)	797
VI.74 Frigyes (Frédéric) Riesz (1880-1956)	798
VI.75 Luitzen Egbertus Jan Brouwer (1881-1966)	799
VI.76 Emmy Noether (1882-1935)	800
VI.77 Wacław Sierpinski (1882-1969)	801
VI.78 George Birkhoff (1884-1944)	802
VI.79 John Edensor Littlewood (1885-1977)	803
VI.80 Hermann Weyl (1885-1955)	805
VI.81 Thoralf Skolem (1887-1963)	806
VI.82 Srinivasa Ramanujan (1887-1920)	807
VI.83 Richard Courant (1888-1972)	808
VI.84 Stefan Banach (1892-1945)	809
VI.85 Norbert Wiener (1894-1964)	811
VI.86 Emil Artin (1898-1962)	812
VI.87 Alfred Tarski (1901-1983)	813
VI.88 Andrei Nikolaevich Kolmogorov (1903-1987)	814
VI.89 Alonzo Church (1903-1995)	816
VI.90 William Vallance Douglas Hodge (1903-1975)	816
VI.91 John von Neumann (1903-1957)	817
VI.92 Kurt Gödel (1906-1978)	819
VI.93 André Weil (1906-1998)	819
VI.94 Alan Turing (1912-1954)	821
VI.95 Abraham Robinson (1918-1974)	822
VI.96 Nicolas Bourbaki (1935-)	823

Part VII The Influence of Mathematics

VII.1 Mathematics and Chemistry	827
VII.2 Mathematical Biology	837
VII.3 Wavelets and Applications	848
VII.4 The Mathematics of Traffic in Networks	862
VII.5 The Mathematics of Algorithm Design	871
VII.6 Reliable Transmission of Information	878
VII.7 Mathematics and Cryptography	887
VII.8 Mathematics and Economic Reasoning	895
VII.9 The Mathematics of Money	910
VII.10 Mathematical Statistics	916
VII.11 Mathematics and Medical Statistics	921
VII.12 Analysis, Mathematical and Philosophical	928
VII.13 Mathematics and Music	935
VII.14 Mathematics and Art	944

Part VIII Final Perspectives

VIII.1 The Art of Problem Solving	955
VIII.2 "Why Mathematics?" You Might Ask	966
VIII.3 The Ubiquity of Mathematics	977
VIII.4 Numeracy	983
VIII.5 Mathematics: An Experimental Science	991
VIII.6 Advice to a Young Mathematician	1000
VIII.7 A Chronology of Mathematical Events	1010
Index	1015

[Return to Book Description](#)

File created: 11/14/2008

Questions and comments to: webmaster@press.princeton.edu
Princeton University Press