HOMEWORK 1, M 331
DUE 2/12/09

Problem 1. Given the functions $g(x) = \ln(x)$, $g(x) = e^{1-x}$, $g(x) = 1/x$, and $g(x) = x\sin(\pi x^2)$ find in each case

(i) a function $f(x)$ whose derivative is $g(x)$.
(ii) Is there another function $h(x)$ different from $f(x)$ whose derivative is $g(x)$? If so, how many functions are there whose derivative is $g(x)$?
(iii) Among all the functions $f(x)$ whose derivative is $g(x)$ find the one which satisfies $f(1) = -1$.

Problem 2. The population of the US was 8.6 Million in 1820 and 40 Million in 1897.

(i) Write down the differential equation for population growth in this case.
(ii) Calculate the number of people in the US according to your model in the year 2003. How well does this growth model work: the actual population of the US in 2003 was 291 Million.
(iii) How long does it take for the population in the US to double according to your growth model?

Problem 3. Calculate the impact velocity if you jump off a 7 foot wall (ignore air drag).

Problem 4. The initial phrasing of the problem was inconsistent; here a new version 2/6/09, 12:00 noon

A skydiver weighing 180 pounds jumps form a height of 5000 feet. Ignore air drag if the parachute is closed. Calculate the drag coefficient needed for the open parachute so that the terminal velocity is no greater then the impact velocity calculated in the previous problem. What is the lowest height at which the skydiver can open the parachute for a save landing.

Problem 5. What is the minimal hight above ground so that an object of mass 10 kg dropped from this height hits the ground at a speed within 5% of its terminal velocity, assuming the drag coeeficient is 2 kg/sec ?

Problem 6. The population dynamics of rabbits in a certain habitat is described by

$$\frac{dP}{dt} = P - 500$$

where $P(t)$ denotes the rabbit population at time t (measured in months), and 500 is the amount of rabbits eaten by preying animals etc per month.

(i) For which initial rabbit population P_0 does the rabbit population stay constant over time?
(ii) For which initial rabbit populations do the rabbits die out?
(iii) Can it also happen that the rabbits just keep growing despite the fact that some are eaten all the time? What initial population is needed for that to happen?

Problem 7. Fish growth in a certain area of the ocean can be modeled by the logistic differential equation. Assume that $r = 0.7$ and the carrying capacity $K = 80\times10^6$ kilograms (the mass of all the fish). If the initial population is $P_0 = 0.25\times K$

find the population (measured in kilograms) 2 years later. Also find the time when the population reaches 3/4 of the carrying capacity.