HOMEWORK 2, M 331
DUE 2/19/09

Problem 1. Consider the ODE

$$y' = (y-1)(y+1)$$

(i) Draw a slope line picture in the (t, y)-plane and draw the graph of the driving term.
(ii) Determine all the equilibrium solutions and characterize them as stable, unstable or semi-stable.
(iii) Determine all the solutions $y(t)$ of the ODE and graph them in the (t, y)-plane.
(iv) Find the solutions $y_1(t)$, $y_2(t)$ and $y_3(t)$ with initial conditions $y_1(0) = -1$, $y_2(0) = 0$ and $y_3(0) = 2$ respectively.

Problem 2. Consider the ODE $y' = y^2$.

(i) What are the equilibrium solutions? Are they stable, unstable or semi-stable?
(ii) Find all solutions $y(t)$ of the ODE.
(iii) Calculate the solution with initial condition $y(0) = 1$ and determine the time s when the solution "blows up", i.e., goes to infinity, i.e., when $\lim_{t\to s} y(t) = \pm\infty$.
(iv) Can you verify (make sense, understand) the statement: **all solutions, but one, explode in finite time**. Which solution does not explode in finite time?

Problem 3. Solve the ODE $y' = \frac{y^2}{t^2-1}$ with initial condition $y(0) = 1$.

Problem 4. Consider the ODE $(y')^2 = y^2 - 1$.

(i) Draw a slope line picture in the (t, y)-plane.
(ii) What are the equilibrium solutions? Are they stable, unstable or semi-stable?
(iii) Find all solutions $y(t)$ of the ODE and graph them in the (t, y)-plane.
(iv) Calculate the solution with initial condition $y(0) = 2$. Can you find solutions to any initial condition $y(0) = c$ where c is an arbitrary number?

Problem 5. Find all solutions to the ODE $y' = 2te^{y-t^2}$. Which of those solutions satsify $y(1) = 1$?

Problem 6. Consider the ODE $y' = \sqrt{y}$.

(i) Find all the solutions of this ODE that satisfy $y(0) = 0$.
(ii) Find all solutions of this ODE which satisfy $y(1) = 0$.
(iii) If you think of ODEs as modeling physical processes, then the initial condition should determine the solution uniquely, one might think.

Problem 7. Solve the ODE $y' = t\cos^2(y)$ and determine all the equilibrium solutions. Which solution satisfies $y(0) = 0$?