

HOMEWORK 3, M 331  
DUE 2/26/09

**Problem 1.** Consider the ODE

$$(2ty^2 + 2y)dt + (2t^2y + 2t)dy = 0$$

- (i) Verify that this equation is exact, i.e., of the form  $Mdt + Ndy = 0$  with  $M_y = N_t$  (where partial derivatives w.r.t. the corresponding variable is denoted by a subscript:  $M_y = \frac{\partial M}{\partial y}$  etc).
- (ii) Find the function  $F(t, y)$  describing the solutions implicitly via  $F(t, y) = C$ .
- (iii) Determine the constant  $C$  so that the solution fulfills  $y(0) = 0$ . Draw this solution somehow.

**Problem 2.** Consider the ODE

$$(ye^{2ty} + t)dt + bte^{2ty}dy = 0$$

- (i) For which value of  $b$  is this ODE exact?
- (ii) Find all solutions of the ODE (possibly implicitly) for this value of  $b$ .

**Problem 3.** Solve (possibly implicitly) the ODE  $(2t - y)dt + (2y - t)dy = 0$  with initial condition  $y(1) = 3$ . Draw the solution curve.

**Problem 4.** Solve the following ODEs by any method you have learned (if no initial condition is specified, find all solutions, possibly implicitly):

- (i)  $ydt + tdy = 0$  with initial condition  $y(1) = 1$ .
- (ii)  $y' = \frac{2t+4y}{3y-4t}$ .
- (iii)  $y' = \frac{2t+3}{2-2y}$  with initial condition  $y(0) = 1$ .
- (iv)  $(9t^2 + y - 1)dt + (t - 4y)dy = 0$  with initial condition  $y(1) = 0$ .

**Problem 5.** Consider the ODE

$$(e^{2t} + y - 1)dt - dy = 0$$

- (i) Show that this ODE is not exact.
- (ii) Find an integrating factor for the ODE.
- (iii) Solve the ODE.

**Problem 6.** Solve the ODE  $ydt + (2t - ye^y)dy = 0$ .