HOMEWORK 5, M 331
DUE 3/12/09

Problem 1. Consider the linear 2nd order homogeneous ODE

$$y'' + 5y' + 6y = 0$$

(i) Assuming there is a solution of the form $y(t) = e^{\lambda t}$ find the condition on λ such that this $y(t)$ solves the ODE.
(ii) Write down the general soultion of the ODE.
(iii) Find the solution which satisfies the initial conditions $y(0) = y'(0) = 1$.
(iv) Can you find a formula for the solution which satisfies the initial condition $y(0) = y_0$ and $y'(0) = v_0$? Once you have such a formula plug in the previous case $y_0 = v_0 = 1$ and check if you got the same answer.

Problem 2. Find all solutions of the ODE $2y'' - 3y' + y = 0$.

Problem 3. Find the solution of the ODE $2y'' + 2y' - 4y = 0$ with initial conditions $y(0) = 0$ and $y'(0) = 1$.

Problem 4. Solve the initial value problem

$$y'' - 4y = 0\,, \qquad y(0) = 1\,, \quad y'(0) = v_0$$

and determine v_0 in such a way that this solution approaches zero as $t \to \infty$.

Problem 5. Consider the inhomogeneous ODE $y'' - 3y' + 2y = 1$.

(i) Guess one solution of this ODE (Hint: what are the constant solutions?)
(ii) Find all solutions of the corresponding homogeneous ODE.
(iii) Find all solutions of the inhomogeneous ODE. What happens to those solutions as $t \to \infty$?
(iv) Find the solution with initial condition $y(0) = y'(0) = 0$.

Problem 6. Consider the ODE $y'' - 4y' + 5y = 0$.

(i) Assuming there is a solution of the form $y(t) = e^{\lambda t}$ determine all λ such that this $y(t)$ solves the ODE.
(ii) Use Euler's Formula $e^{ix} = \cos x + i \sin x$ and the superposition principle (applied to the 2 solutions found in (i)) to show that $e^{2t} \sin t$ and $e^{2t} \cos t$ are solutions of this ODE.
(iii) Determine the solution which satisfies the initial condition $y(0) = 1$ and $y'(0) = 0$.

Problem 7. Consider the inhomogeneous ODE $y'' + 6y' + 13y = 5$.

(i) Find a solution of this ODE (compare with Problem 5).
(ii) Find all solutions to the corresponding homogenous ODE.
(iii) Find all solutions to the inhomogeneous ODE. How do the solutions behave for $t \to \infty$?
(iv) Find the solution with initial conditions $y(0) = 0$ and $y'(0) = 1$.