

HOMEWORK 8, M 331
DUE 4/9/09

Problem 1. Consider the inhomogeneous ODE

$$t^2 y'' - 2y = 3t^2 - 1.$$

- (i) Show that $y_1(t) = t^2$ and $y_2(t) = 1/t$ are solutions to the homogeneous ODE.
- (ii) Verify that y_1 and y_2 are independent solutions.
- (iii) Find a particular solution of the ODE using the variation of parameter method.

Problem 2. Consider the inhomogeneous ODE

$$t^2 y'' - 3ty' + 4y = t^2 \ln t.$$

- (i) Show that $y_1(t) = t^2$ and $y_2(t) = t^2 \ln t$ are solutions to the homogeneous ODE.
- (ii) Verify that y_1 and y_2 are independent solutions.
- (iii) Find a particular solution of the ODE using the variation of parameter method.

Problem 3. Find the general solution of the inhomogeneous ODE

$$y'' + 4y' + 4y = \frac{1}{t^2} e^{-2t}.$$

Problem 4. A mass of 100 grams stretches a spring 3 cm (this determines the spring constant k , since restoring force equals k times displacement by Hooke's Law; be aware that force is mass times gravitational constant; make sure you get the units right). If the mass is set in motion from its equilibrium position with a downwards velocity of 10 cm/sec, and if there is no damping, determine the position of the mass at any time t . When does the mass first return to its equilibrium position?

Problem 5. Consider the forced but undamped system described by the initial value problem

$$y'' + y = 3 \cos(\omega t), \quad y(0) = y'(0) = 0.$$

- (i) Find the solution $y(t)$ for $\omega \neq 1$.
- (ii) Plot the solution $y(t)$ versus t for $\omega = 0.7$, $\omega = 0.8$, and $\omega = 0.9$. Describe how $y(t)$ changes as ω varies in this interval. What happens as ω takes on values closer and closer to 1? Note that the natural frequency of the unforced system is $\omega_0 = 1$.