PRACTICE FINAL, M 331, FOR FINAL EXAM ON TUESDAY 5/19/09, 1:30 PM

You can use any method to solve the ODEs below (except computer software such as mathematica etc). Show all work.

Problem 1. Find the general solution to the ODE

$$y'' - 2y' + y = \frac{e^t}{t}$$

Problem 2. Consider the ODE

$$y' = \begin{pmatrix} 1 & 1 \\ -4 & 1 \end{pmatrix} y$$

(i) Find the solution with initial condition $y(0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

(ii) Draw a picture of the general behavior of the solutions and characterize whether the origin is a source, sink, saddle or spiral point.

Problem 3. Consider the matrix ODE

$$y' = \begin{pmatrix} 1 & 3 \\ 3 & 1 \end{pmatrix} y$$

(i) Find the general solution.

(ii) Sketch the general behavior of the solutions and characterize whether the origin is a source, sink, saddle or spiral point.

Problem 4. Solve the linear ODE

$$y'' + 9y = \delta(t-1)$$

with initial conditions $y(0) = 0$ and $y'(0) = 1$.