1. a: Write the system of equations below as a matrix equation.
   b: Find all solutions using Gauss elimination:

   \[-x - 2y - 4z = 0, -x + 3y + z = -5, 2x + y + 5z = 3.\]

2. What does it mean for a vector to be in the image of a matrix \(A\).
   Let \(A\) be the matrix:
   \[
   \begin{pmatrix}
   1 & 2 & 5 \\
   -2 & 0 & -2 \\
   3 & -1 & 1 \\
   \end{pmatrix}
   \]
   Is \[
   \begin{pmatrix}
   1 \\
   2 \\
   -1 \\
   \end{pmatrix}
   \]
   an element of the image of \(A\)?
   Why?

3. Define what it means for a set \(s\) to be a basis of a subspace \(V \subset \mathbb{R}^n\). Let
   \[
   A = \begin{pmatrix}
   1 & 2 & 3 & -1 \\
   -1 & 0 & 1 & -1 \\
   -1 & 4 & 3 & -5 \\
   \end{pmatrix}
   \]
   Give a set of vectors that span \(\text{im}(A)\) and that are independent.

4. Let \(A\) be a \(n \times m\) matrix, so \(A\) gives a linear transformation from \(\mathbb{R}^m\) to \(\mathbb{R}^n\). Let \(x_1, x_2 \in \mathbb{R}^m\). Assume that \(A(x_1) = 0\) and that \(A(x_2) = b\).
   Explain why \(A(x_1 + x_2) = b\).

5. Let \(A\) be a two by two matrix that rotates by angle \(2\pi/6\). a: Find \(A\).
   b: Give a geometric explanation why \(A^6 = I_2\). Here \(I_2\) denotes the 2 by 2 identity matrix.

6. Solve the equation
   \[
   \begin{pmatrix}
   1 & 0 & -2 \\
   0 & 0 & 2 \\
   2 & 1 & -1 \\
   \end{pmatrix}
   \begin{pmatrix}
   x_1 \\
   x_2 \\
   x_3 \\
   \end{pmatrix}
   =
   \begin{pmatrix}
   1 \\
   -2 \\
   0 \\
   \end{pmatrix}
   \]
   for \(x = \begin{pmatrix}
   x_1 \\
   x_2 \\
   x_3 \\
   \end{pmatrix}\) by finding the inverse of the given matrix.

7. Compute the product \(AB\) of the two matrices \(A, B\) given below, if possible. If it is not possible say why it is not possible.
   \[
   A = \begin{pmatrix}
   1 & 2 \\
   -1 & 0 \\
   3 & -2 \\
   \end{pmatrix}
   \]
   \[
   B = \begin{pmatrix}
   -1 & 0 & 0 \\
   4 & 0 & 7 \\
   \end{pmatrix}
   \]
8. Let $W$ denote the set of vectors orthogonal to the vector $\begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$. Find a basis of $W$.

9. Consider the matrix

$$A = \begin{pmatrix} 1 & 0 & 2 \\ -1 & 2 & 0 \\ 1 & 1 & 3 \\ -2 & 1 & -3 \end{pmatrix}.$$ 

Let $b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix}$. Find equations in $b_1, b_2, b_3, b_4$ so that the equation $Ax = b$ can be solved. Find a basis of the image of $A$.

10. True or False. It is necessary to give an explanation.

   a: The image of a 3 by 4 matrix $A$ is a subspace of $\mathbb{R}^4$.
   
   b: If the kernel of a matrix $A$ consists of the zero vector only, then the column vectors of $A$ must be linearly independent.
   
   c: If $u, v, w$ are in a subspace of $\mathbb{R}^n$, then so is $3u - v - w$.
   
   d: The function
   $$T: \begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} x - y \\ x + y \end{pmatrix}$$
   
   is a linear transformation.
   
   e: If $A$ is an invertible $n \times n$ matrix, then the reduced row echelon form of $A$ is $I_n$.
   
   f: The formula $AB = BA$ holds for all $n \times n$ matrices $A$ and $B$.
   
   g: If the $4 \times 4$ matrix $A$ has rank 4, then any linear system with coefficient matrix $A$ will have a unique solution.

11. An $n \times n$ matrix $M$ defines a linear map from $\mathbb{R}^n$ to $\mathbb{R}^n$. Note that $M^2$ also defines a linear map from $\mathbb{R}^n$ to $\mathbb{R}^n$.

   a: Show that if $v \in \text{im}(M^2)$, then $v \in \text{im}(M)$, meaning we have the inclusion of subspaces $\text{im}(M^2) \subset \text{im}(M)$.
   
   b: Find a $3 \times 3$ matrix $M$ with $\text{im}(M) \neq \text{im}(M^2)$. 