PRACTICE WITH DIFFERENTIAL FORMS

The following problems concern these differential forms on the domain $R = \mathbb{R}^4 = \{(t, x, y, z)\}$:

$$\alpha := x\ dy + z\ dt \in \Omega^1(R), \quad \beta := dt \wedge dx + dy \wedge dz \in \Omega^2(R), \quad \gamma := \sin t\ dx \wedge dy \wedge dz \in \Omega^3(R).$$

Problem 0.1. *Find their exterior derivatives* $d\alpha$, $d\beta$, *and* $d\gamma$.

Problem 0.2. *Compute their exterior squares:* $\alpha \wedge \alpha$, $\beta \wedge \beta$, *and* $\gamma \wedge \gamma$. *Why are some of these automatically* 0*?*

Problem 0.3. *Compare the exterior products* $\alpha \wedge \beta$, $\alpha \wedge \gamma$, $\beta \wedge \gamma$ *with* $\beta \wedge \alpha$, $\gamma \wedge \alpha$, $\gamma \wedge \beta$.

Problem 0.4. *Work out the exterior derivatives* $d(\alpha \wedge \beta)$ *and* $d(\beta \wedge \alpha)$, *both directly and by using the Leibniz formula for the wedge product of forms. How do they compare?*

Problem 0.5. *Compute the exterior derivative* $d(\alpha \wedge \gamma)$. *In retrospect, can you explain why you should have expected this result?!*

Rob Kusner, Mathematics & Statistics, University of Massachusetts, Amherst, MA 01003
E-mail address: profkusner@gmail.com

Date: 04 Nov 2019.