5e.pdf

Warm-up Problems

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f & g are functions on $\mathbb{R}^3$, and $\vec{v}$ & $\vec{w}$ are vector fields on $\mathbb{R}^3$, in the following True/False problems – please give a proof (if true) or counterexample (if false):

1. $\nabla(fg) = (\nabla f)g + f(\nabla g)$
2. $\nabla(\vec{v}\cdot\vec{w}) = (\nabla\cdot\vec{v})\vec{w} + \vec{v}(\nabla\cdot\vec{w})$
3. $\nabla\cdot(f\vec{v}) = \nabla f\cdot\vec{v} + f\,\nabla\cdot\vec{v}$
4. $\nabla\cdot(\vec{v}\times\vec{w}) = (\nabla\times\vec{v})\cdot\vec{w} - \vec{v}\cdot(\nabla\times\vec{w})$
5. $\nabla\times(f\vec{v}) = \nabla f\times\vec{v} + f\,\nabla\times\vec{v}$
6. There is a function h on $\mathbb{R}^3$ such that $\nabla f\times\nabla g = \nabla h$. (Hint: work out a formula for $\nabla\times(\vec{v}\times\vec{w})$, which is the last of the basic Leibniz/product formulas we may need. Then ask yourself: when is a vector field, like $\nabla f\times\nabla g$, a gradient?)

[You should also practice your path integrals!]