

## PRACTICE WITH DIFFERENTIAL FORMS

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The following problems concern these differential forms on the domain  $R = \mathbb{R}^4 = \{(t, x, y, z)\}$ :  
 $\alpha := x dy + z dt \in \Omega^1(R)$ ,  $\beta := dt \wedge dx + dy \wedge dz \in \Omega^2(R)$ ,  $\gamma := \sin t dx \wedge dy \wedge dz \in \Omega^3(R)$ .

**Problem 0.1.** Find their exterior derivatives  $d\alpha$ ,  $d\beta$ , and  $d\gamma$ .

**Problem 0.2.** Compute their exterior squares:  $\alpha \wedge \alpha$ ,  $\beta \wedge \beta$ , and  $\gamma \wedge \gamma$ . Why are some of these automatically 0?

**Problem 0.3.** Compare the exterior products  $\alpha \wedge \beta$ ,  $\alpha \wedge \gamma$ ,  $\beta \wedge \gamma$  with  $\beta \wedge \alpha$ ,  $\gamma \wedge \alpha$ ,  $\gamma \wedge \beta$ .

**Problem 0.4.** Work out the exterior derivatives  $d(\alpha \wedge \beta)$  and  $d(\beta \wedge \alpha)$ , both directly and by using the Leibniz formula for forms. How do they compare?

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