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DEPARTMENT OF MATHEMATICS AND STATISTICS
UNIVERSITY OF MASSACHUSETTS
FINAL EXAM – MATH 671 – TOPOLOGY
ROB KUSNER – DECEMBER 2000

Work alone. Answer all questions. Justify your answers.

Return completed exam papers to Rob (or his mailbox) before 23 December 2000.

1. Consider a few of the "other" topologies on $\mathbb{R}$:
(i) trivial,
(ii) Zariski (= finite complement),
(iii) discrete.
For each of these topologies check whether the interval $[a, b]$ is

(a) closed, (b) connected, (c) compact?

2. Show that every map from the circle $S^1 \to \mathbb{R}^n$ extends to a map from the disk $D^2 \to \mathbb{R}^n$.

3. Consider the "infinite tee" in the plane, i.e., the subspace $T \subset \mathbb{R}^2$ defined by the union of the x-axis and the negative y-axis. Show that T has a one-point compactification $\hat{T}$. To which letter of the Greek alphabet is $\hat{T}$ homeomorphic?

4. Decide whether the subspaces A, B and C of $\mathbb{R}^2$ suggested by the corresponding letters of the Roman alphabet, are homeomorphic to each other or not. If a pair of spaces is not homeomorphic, can one be embedded in the other?

5. Let X and Y be connected and let $A \subset X$ and $B \subset Y$ be proper subsets. Show that $(X \times Y) - (A \times B)$ is connected.

6. Let X be any space, and let Y be a compactly generated, Hausdorff space. Show that a proper bijective map

$$f : X \to Y$$

is a homeomorphism.