Homework 11, M 331
due 11/26/13

Please hand in your homework before class, have it neatly written (the grader will not decipher your notes but instead will give you zero points), stapled, with your name and student ID on top.

Solve the following initial value problems by using the Laplace transform.

**Problem 1.** Consider the undamped oscillator

\[ y'' + 4y = r(t) \]

with initial conditions \( y(0) = 0 \) and \( y'(0) = 2 \), where the external forcing term is given by

\[
r(t) = \begin{cases} 
0 & \text{if } 0 \leq t < 2 \\
t & \text{if } t \geq 2
\end{cases}
\]

(i) Express \( r(t) \) as a product of a (shifted) step function and the function \( t \).
(ii) Calculate the Laplace transform \( \mathcal{L}[y](s) \).
(iii) To apply the Laplace table formula 13 rewrite your \( r(t) \) from (i) using the trick

\[
tu_a(t) = (t-a)u_a(t) + au_a(t)
\]

(iv) Calculate \( y(t) \).

**Problem 2.** Find the solution of the inhomogeneous ODE

\[ y'' + 3y' + 2y = \delta_5(t) \]

with initial conditions \( y(0) = 0 \) and \( y'(0) = 1 \). (Note that \( \delta_5(t) = \delta(t - 5) \) so that formula 17 of the Laplace table applies).