This is a collection of problems you should be able to do without much difficulty. A (small) subset of similar problems will be on the exam. The solutions will be discussed in the TAs review sessions.

You can hand in a neat writeup of the solutions for extra credit on Thursday 10/17 before the exam. Later hand-ins cannot be accepted for extra credit.

Problem 1. Consider the differential equation

\[ y' = (y - 4)(y - 1) \]

(i) Draw a slope line diagram for the ODE, indicating the equilibria and the generic behavior of the non-equilibrium solutions.

(ii) Draw a space diagram for the ODE, indicating the equilibria and the behavior of the non-equilibrium solutions.

(iii) Categorize the equilibria as stable, unstable or semi-stable.

(iv) Find the solution of the ODE that satisfies the initial condition \( y(0) = 0 \).

Problem 2. Calculate the time it takes for a mass of 80 kg dropped from a height of 50 meters to reach the ground. Assume the drag coefficient to be 1 and that drag is modeled proportional to velocity.

Problem 3. How high does a stone weighing 5 ounces launched from the ground with initial velocity 5 ft per second fly, and how long will this stone be in the air? Ignore air drag.

Problem 4. Find the solutions to the ODE \( y' + t^2 y = 5t^2 \) with initial conditions \( y(0) = 0 \) and \( y(0) = 5 \) respectively.

Problem 5. Find the general solution to the ODE

\[ y' - \frac{3}{7} y = t^3. \]

Problem 6. Solve the initial value problem

\[
\begin{align*}
\frac{dy}{dt} &= 2y^2 + ty^2 \\
y(0) &= -1/2
\end{align*}
\]

and determine whether and when the solution has a vertical asymptote.

Problem 7. Find the solution of the ODE \( y'' + 16y = 0 \) with initial condition \( y(0) = 1 \) and \( y'(0) = 2 \).

Problem 8. Find the solution of the ODE \( y'' - 4y = 1 \) with initial condition \( y(0) = 0 \) and \( y'(0) = 0 \).

Problem 9. Find the general solution of the ODE \( y'' + y' - 2y = 0 \).

Problem 10. Find the solution of the ODE \( y'' - 4y' + 5y = 0 \) with initial condition \( y(0) = 1 \) and \( y'(0) = 0 \).